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TAPPED DELAY LINE REALIZATIONS OF FREQUENCY PERIODIC FILTERS AND THEIR APPLICATION TO LINEAR FM PULSE COMPRESSION

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R. Manasse

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DIRECTORATE OF RADAR AND OPTICS

ELECTRONIC SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts

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ABSTRACT

It is shown that a linear network having an amplitude and phase response which is a periodic function of frequency can be synthesized with a tapped delay line with amplitude and phase weightings on each tap. The theory of this technique for the realization of frequency periodic filters is developed. The example which motivates the discussion of this problem is the use of a single frequency periodic filter to replace a bank of complex dispersive subpulse networks employed in a large time-bandwidth product linear FM pulse compression network. The availability of high quality tapped quartz delay lines and the ease with which amplitude and phase adjustments can be made on each tap appear to make this technique attractive for a number of future applications.

PREFACE

The material for this paper was prepared approximately a year and one half ago with the expectation that it would eventually form one section of a considerably larger report on linear FM pulse compression. Since that time the material for the larger report has grown and evolved into several papers, two of which were presented at the recent Pulse Compression Symposium at RADC. It is more than timely therefore to publish this material at this time and, except for minor editorial revisions, this TM reproduces the draft version of this paper prepared earlier.

TAPPED DELAY LINE REALIZATIONS OF FREQUENCY PERIODIC FILTERS AND THEIR APPLICATION TO LINEAR FM PULSE COMPRESSION

In this paper we shall see how the desired periodic filter response can be realized with the aid of a high quality tapped delay line having amplitude and phase weightings on the output of each tap. Such a filter has a frequency response which is periodic in frequency with a period equal to the reciprocal of the delay line tap spacing.

In our case, we are interested in synthesizing a set of dispersive networks which are all identical except for a center frequency displacement. If this tapped delay line simulates the desired frequency response over a frequency band, then the repetitive character of the network in frequency enables us to use the filter at a number of frequencies simultaneously. Thus we are able to replace a whole set of dispersive networks by one tapped delay line.

Before considering the reasoning which leads us to the use of a tapped delay line, we will review briefly the basic properties of the complex notation which is used to characterize the response of linear networks.*

Consider a linear time-invariant network which is characterized by an impulse response h(t). We let x(t) be the real waveform input to the filter, and y(t) be the real waveform at the output of the filter. See Figure 1.

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* For a discussion of the complex representation of real waveforms, see P. M. Woodward, "Probability and Information Theory, with Applications to Radar" (McGraw-Hill, 1953), and D. Gabor, Journal Institute of Elect. Engineers (Pt. III), 93, p. 429, 1946. See also, J. Dugundji, "Envelopes and Pre-Envelopes of Real Waveforms", Vol. IT-4, PGIT, March, 1958.

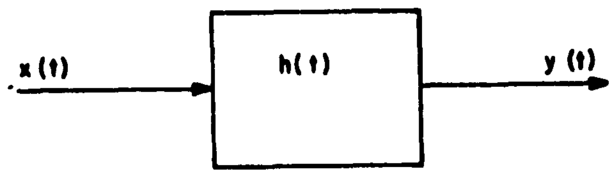

FIGURE 1

y(t) is related to x(t) and h(t) by a convolution integral.

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

All realizable filters satisfy h(t) = 0 for $t < 0$, so that the convolution can equally well be written

$$y(t) = \int_{-\infty}^{t} h(t-\tau)x(\tau)d\tau$$

Convolution in the time domain corresponds to multiplication in the frequency domain, and therefore

$$Y(f) = H(f)X(f) \quad \left[\text{or } Y(\omega) = H(\omega)X(\omega),\ \omega = 2\pi f\right]$$

where Y(f), H(f) and X(f) are Fourier transforms of y(t), h(t) and x(t), respectively.

Usually, in circuit theory, the response of an electrical network is characterized by its effect on complex time waveforms (e.g., $e^{j\omega t}$, $j = \sqrt{-1}$) rather than real time waveforms (e.g., $\cos \omega t$). The reason for this, of course, is that any real time waveform must have a Fourier transform which is conjugate symmetric about zero frequency and therefore any real waveform is completely characterized by its frequency function for positive frequencies alone: The frequency function for negative frequencies can be mapped to zero without destroying any information about the time waveform, and this is exactly the procedure used to obtain the complex representation which proves to be notationally convenient and leads to algebraic simplifications. $X_c(f)$, the complex (frequency) representation of $X(f)$, is defined simply as

$$X_c(f) = \begin{cases} 0 \quad , & f < 0 \\ X(f), & f > 0 \end{cases}$$

$Y_c(f)$ is similarly defined. The frequency response of the network is then written

$$Y_c(f) = H(f)X_c(f)$$

The complex time representation of $x(t)$, written $x_c(t)$, is the inverse Fourier transform of $X_c(f)$. The real waveform can always be obtained from its complex representation by taking twice the real part. This complex representation sets up a one-to-one correspondence between real and complex waveforms. In the frequency domain the correspondence is obtained by mapping the frequency function to zero for negative frequencies, and leaving the frequency function for positive frequencies unperturbed. In the time domain the correspondence between real and complex waveforms is set up using Hilbert transforms. $x_c(t)$, the complex representation of $x(t)$, is given by

$$x_c(t) = \frac{1}{2}\, x(t) + \frac{1}{2}\, j\mathcal{H}x(t)$$

where $\mathcal{H}x(t) \equiv$ Hilbert transform of $x(t)$. We find, taking the Fourier transform of both sides, that $\mathcal{H}$ must satisfy

$$X_c(f) = \frac{1}{2} X(f) + \frac{1}{2} j F\left[\mathcal{H} x(t)\right] ,$$

(F denotes Fourier transform)

or

$$F\left[\mathcal{H} x(t)\right] = \begin{cases} -jX(f), & f > 0 \\ jX(f), & f < 0 \end{cases}$$

It is seen that $\mathcal{H}$ introduces a multiplication by $-j$ for positive frequencies and multiplication by $+j$ for negative frequencies. In the time domain this operation corresponds to convolution with the time function $1/\pi t$, so

$$\mathcal{H} x(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)} d\tau$$

which is the usual formula for the Hilbert transform. It is as though $x(t)$ had been passed through a linear filter with impulse response $1/\pi t$. However, this filter is not realizable because the impulse response is not zero for t less than zero. $\mathcal{H}$ can be viewed in the frequency domain as an infinite-bandwidth 90° phase shifter. If we allow arbitrarily long time delays in the impulse response, it should be possible to realize $\mathcal{H}$ to any desired degree of accuracy. Fortunately in the synthesis of filters, time delays are usually of no consequence.

$\mathcal{H}$ has a number of useful properties, but we shall need only a few of them.

$$\mathcal{H}(\cos \omega t) = \sin \omega t \quad (\omega = 2\pi f)$$

$$\mathcal{H}(\sin \omega t) = -\cos \omega t$$

$$\mathcal{H}^2 x(t) = -x(t)$$

$$\mathcal{H}(\text{real function}) = \text{real function}$$

The first two relations are easily proved by mapping $\cos\omega t$ and $\sin\omega t$ to their complex representations, multiplying by $-j$, and taking twice the real part. The third relation results from the fact that $\mathcal{H}$ applied twice converts $X(f)$ into $-X(f)$. The fourth relation is true because $\mathcal{H}$ preserves conjugate symmetry of the frequency function.

There is another important relationship which we will need. Consider the following function

$$x(t) = u(t)\cos\omega t$$

where $u(t)$ has a frequency function which vanishes outside the interval $(-\omega,\omega)$. We can obtain the Hilbert transform of $x(t)$ by replacing the right-hand side of the equation by its complex representation, multiplying by $-j$, and taking twice the real part.

The complex representation of $x(t)$ is given by

$$x_c(t) = \frac{1}{2}\,u(t)e^{j\omega t}$$

The Fourier transform of this function, which can be expressed as a convolution of the respective frequency functions, is zero for negative frequencies. Twice the real part of this function equals $x(t)$. Therefore the function must indeed be the complex representation of $x(t)$.

Multiplying the complex representation by $-j$, and taking twice the real part, we have

$$\mathcal{H}\left[u(t)\cos\omega t\right] = 2\mathcal{R}e\left[\frac{-j}{2}\,u(t)e^{j\omega t}\right] = u(t)\sin\omega t$$

Similarly

$$\mathcal{H}\left[u(t)\sin\omega t\right] = -u(t)\cos\omega t$$

Let us now proceed to synthesize, using the above relationships, a function of time whose frequency function is a repetitive version of some desired (non-periodic) function. Consider a network characterized by an impulse response h(t) and a frequency transfer function H(f) with the desired amplitude and phase response over a band of frequencies W_F. Since H(f) is to be used only over the band W_F, we can set H(f) equal to zero outside this band, as shown in Figure 2.

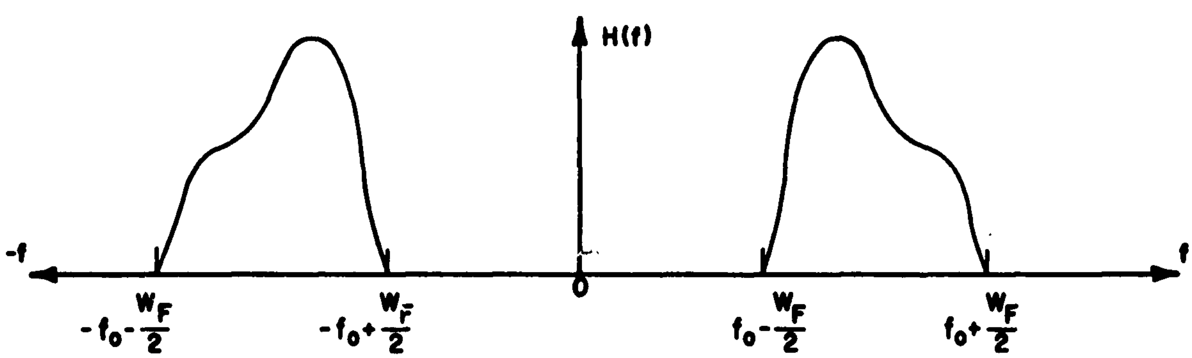

FIGURE 2

H(f) is the Fourier transform of a real function, and therefore it must be conjugate symmetric. If it were not for the image component of H(f) at $-f_o$, we could form the Rep_{W_F} H(f) in order to obtain a periodic function of frequency.† The desired result can be obtained by carrying out the following steps:

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† For discussion of the Rep and Comb operations, see P. M. Woodward, loc. cit., p. 28.

1. Replace h(t) by $h_c(t)$, so that the frequency transfer function is zero for $f<0$.

$$h(t) \rightarrow h_c(t) = \frac{1}{2} h(t) + \frac{1}{2} j \mathcal{H} h(t)$$

2. Make the resulting frequency function periodic by performing Rep_{W_F} on $H_c(f)$. As Woodward shows, the operation Rep_{W_F} in the frequency domain corresponds to the operation $(1/W_F)\ \mathrm{Comb}_{1/W_F}$ in the time domain. The result is, omitting constants of proportionality,

$$\mathrm{Comb}_{T_F}\left[h(t)\right] + j\ \mathrm{Comb}_{T_F}\left[\mathcal{H} h(t)\right] \quad (T_F = 1/W_F)$$

This impulse response as it stands is not satisfactory because it is not real. We must make the frequency function of this waveform conjugate symmetric by mapping the frequency spectrum to zero for negative frequencies and taking the real part of the resulting time waveform.

3. We map the negative frequency part of the frequency function of the above time waveform to zero. For any time waveform (real or complex) this result can be obtained by adding to it j times the Hilbert transform of the time waveform.

The result is

$$\mathrm{Comb}_{T_F}\left[h(t)\right] + j\ \mathrm{Comb}_{T_F}\left[\mathcal{H} h(t)\right]$$

$$+ j\mathcal{H}\mathrm{Comb}_{T_F}\left[h(t)\right] - \mathcal{H}\mathrm{Comb}_{T_F}\left[\mathcal{H} h(t)\right]$$

4. The above waveform has a frequency function with the desired repetitive character for positive frequencies. Because the frequency function is zero for negative frequencies, the waveform is the complex representation of a real waveform which is obtained by taking twice the real part. Ignoring constants of proportionality, as before, we have the desired impulse response h'(t)

$$h'(t) = \mathrm{Comb}_{T_F}\left[h(t)\right] - \mathcal{H}\,\mathrm{Comb}_{T_F}\left[\mathcal{H}h(t)\right]$$

In order to see how this impulse response can be realized with a tapped delay line, let us consider the Comb operation. The Comb operation, as defined by Woodward, multiplies the time waveform by an infinite series of uniformly spaced delta functions.

$$\mathrm{Comb}_{T_F}\left[h(t)\right] = h(t)\sum_{-\infty}^{\infty}\delta(t-kT_F)$$

$$= \sum_{-\infty}^{\infty} h(kT_F)\,\delta\,(t-kT_F)$$

where the summations are over the index k. This impulse response consists of a weighted sequence of δ- functions and it can be realized with a tapped delay line where the output of the kth tap is weighted with amplitude $h(kT_F)$. Writing out the expression for h', we have

$$h'(t) = \sum_{-\infty}^{\infty} h_k\delta(t-kT_F) - \mathcal{H}\left[\sum_{-\infty}^{\infty}\hat{h}_k\delta(t-kT_F)\right]$$

where we have abbreviated $h_k = h(kT_F)$ and $\hat{h}_k = \mathcal{H}h(kT_F)$.

Regarding the problem of realizability, the first term in the expression for h'(t) will be zero for negative t, but the second term in the expression for h'(t) is not necessarily zero for negative t. The reason for this is that the Hilbert transform of a function which is zero for negative t yields a function of time which is not, in general, zero for negative t. We have seen earlier that the Hilbert transform operation is equivalent to passing the waveform through a linear filter with non-realizable impulse response $1/\pi t$ and so this result is not surprising. However, as mentioned earlier, the Hilbert transform can be closely approximated by a realizable filter if sufficiently large time delays are allowed. For example, consider a filter with an impulse response $\left[\pi(t-T)\right]^{-1}$ for $t \geq 0$, and zero for $t < 0$. This function is plotted in Figure 3.

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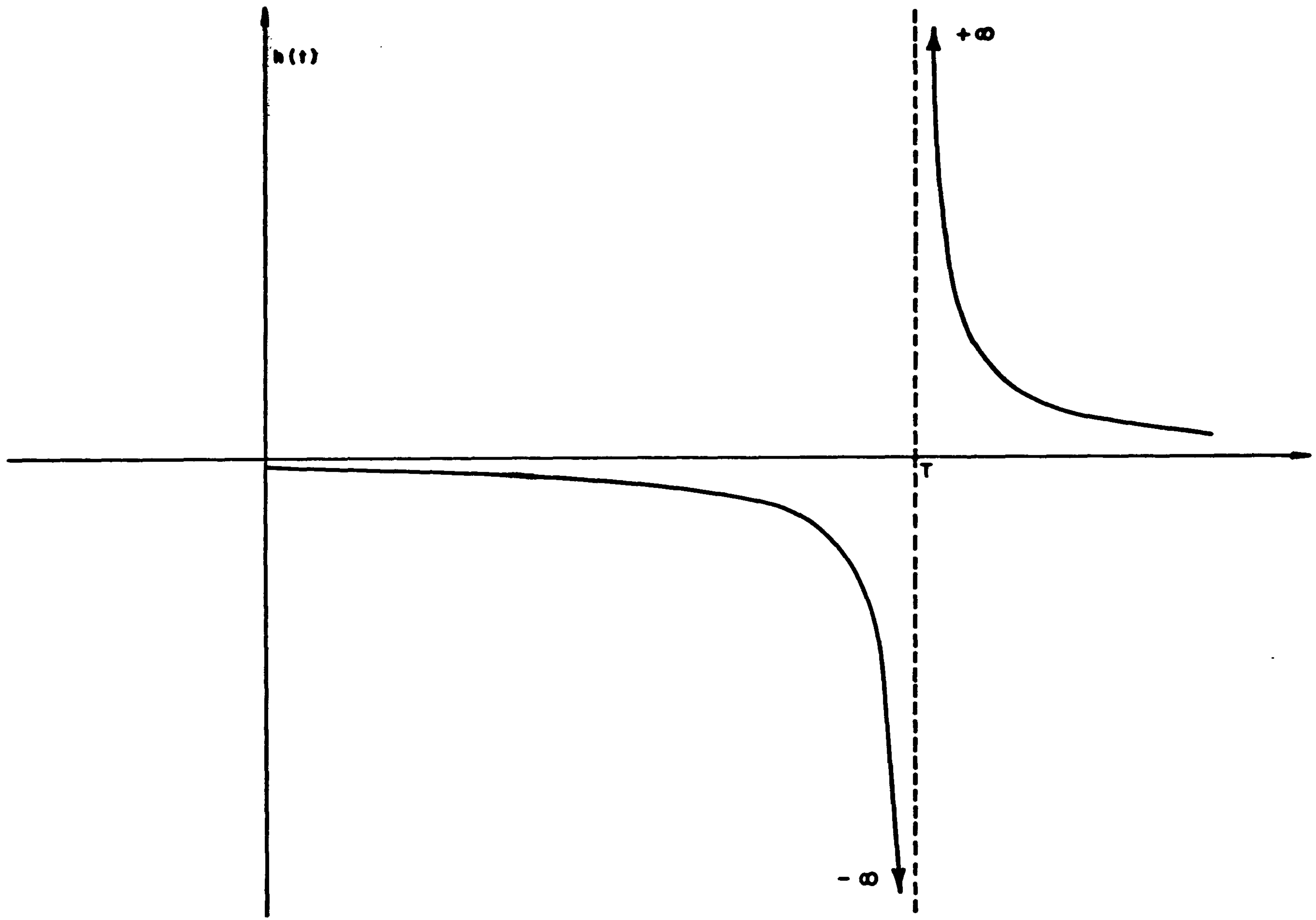

FIGURE 3

This impulse response is a truncated version of a Hilbert transformer in series with a time delay T, the effects of the truncation becoming smaller as T becomes larger. We assume in the discussion which follows that this type of approximation is used for the Hilbert transform wherever it occurs and that sufficient time delay is introduced where necessary to maintain realizability of the required impulse responses. We assume, in particular, that h(t) has been delayed sufficiently so that both h_k and $\hat{h}_k$ are zero for $k < 0$.

The tapped delay line realization of h'(t) is shown in Figure 4.

As we have pointed out earlier, the $-\mathcal{H}$ operation gives a broad-band $+90^{\circ}$ phase shift. A somewhat more practical scheme may be to use at the outputs of the adder channels two broadband $\pm 45^{\circ}$ phase shifters as shown in Figure 5. Here a sufficient (but equal) time delay is assumed to be included in both phase shifters so that they can be accurately realized over the frequency band of interest.

In effect, we can view the operation on the output of each delay line tap as an amplitude weighting a_k and a phase shift ϕ_k, where

$$h_k = a_k \cos \phi_k$$

$$\hat{h}_k = a_k \sin \phi_k$$

or

$$a_k = \left[h_k^2 + \hat{h}_k^2\right]^{\frac{1}{2}}$$

$$\phi_k = \tan^{-1}(\hat{h}_k/h_k)$$

It is useful to work out the expression for the frequency response of the tapped delay line, H'(f), in terms of the coefficients a_k and ϕ_k. H'(f) is found by taking the Fourier transform of the expression obtained earlier for h'(t).

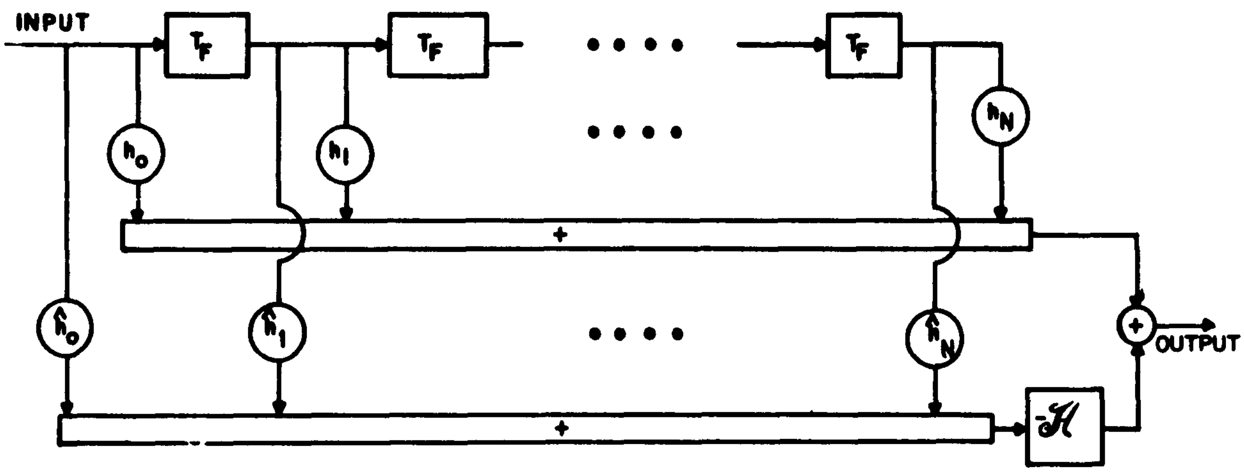

FIGURE 4

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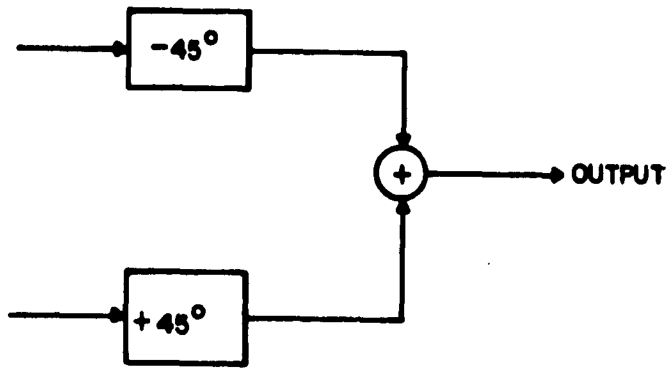

FIGURE 5

$$H'(f) = \int_{-\infty}^{\infty} h'(t)e^{-j2\pi ft}\,dt$$

$$= \sum_{-\infty}^{\infty} h_k e^{-j2\pi fkT_F} + j\sum_{-\infty}^{\infty} \hat{h}_k e^{-j2\pi fkT_F}$$

$$= \sum_{-\infty}^{\infty} a_k e^{j\phi_k} e^{-j2\pi fkT_F}$$

As expected, this function is periodic in frequency with period $W_F = 1/T_F$. Noting the orthogonality relation

$$\int_{f_o-\frac{W_F}{2}}^{f_o+\frac{W_F}{2}} e^{j2\pi f(m-k)T_F}\,df = \left\{\begin{matrix} W_F, \text{ if } m = k \\ 0\ , \text{ if } m \neq k \end{matrix}\right\}$$

where m and k are integers, the expression for H'(f) can be used to find a_k and ϕ_k in terms of H'(f).

$$a_k e^{j\phi_k} = \int_{f_o - \frac{1}{2} W_F}^{f_o + \frac{1}{2} W_f} H(f) e^{j2\pi f k T_F} \, df$$

The prime has been left off the H(f) because H'(f) = H(f) over the region of integration (again ignoring constants of proportionality). Taking the real and imaginary parts of both sides, we obtain

$$h_k = \mathcal{R}e\,(a_k e^{j\phi_k}) = \mathcal{R}e \int_{f_o - \frac{1}{2} W_F}^{f_o + \frac{1}{2} W_F} H(f) e^{j2\pi f k T_F} \, df$$

$$\hat{h}_k = \mathcal{I}m(a_k e^{j\phi_k}) = \mathcal{I}m \int_{f_o - \frac{1}{2} W_F}^{f_o + \frac{1}{2} W_F} H(f) e^{j2\pi f k T_F} \, df$$

Thus it is seen that the required tap weightings, h_k and $\hat{h}_k$, can be easily computed from the required H(f).

The tapped line has a frequency response which is equal to the desired frequency response H(f) in the band of interest, and is periodic in frequency with period equal to the reciprocal of the delay line tap spacing. The behavior of the network frequency response over the band of interest depends only on these tap weightings. Since these tap weightings can be obtained from a set of potentiometer adjustments, the response of the line can be changed easily by readjusting these pots. With the aid of a high quality quartz delay line having many taps, one should be able to synthesize a very complicated network characteristic.

It should be noted that this tapped delay line method ot filter synthesis is perfectly general; its use is not restricted to the synthesis of dispersive networks only.

Suppose we wish to utilize the characteristics of this filter over only one of these frequency periods. The situation is shown schematically in Figure 6. $H_s(f)$ denotes the frequency response of a band selection filter.

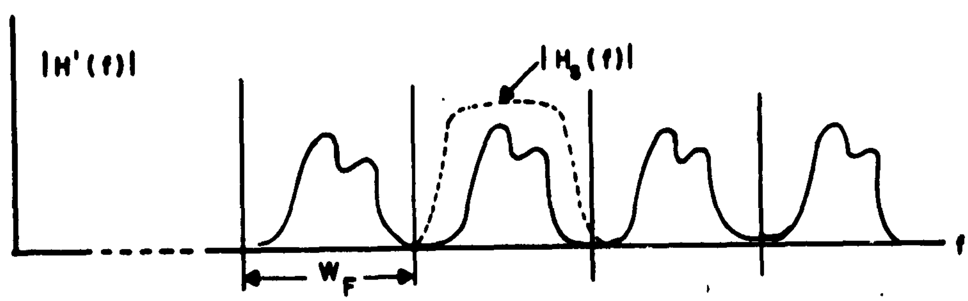

FIGURE 6

In the above plot only the amplitude of the frequency response has been shown. A similar plot could be shown indicating the repetitive phase shift characteristics of the network. That portion of the frequency response which we desire to use can be extracted by means of the band selection filter $H_s(f)$ with a linear phase and a constant amplitude characteristic over the region in which $H'(f)$ differs appreciably from zero, but which drops off sufficiently rapidly to exclude responses from neighboring bands.

More generally, we can consider the desired H(f) to be decomposed into the product $H'(f) \cdot H_s(f)$, where H'(f) is realized with a tapped delay line and $H_s(f)$ represents the band selection filter. In this manner the non ideal characteristics of $H_s(f)$ can be compensated somewhat by adjustment of H'(f). Thus the burden of accurately approximating H(f) can be shared between both H'(f) and $H_s(f)$, though most of the burden will probably still fall on H'(f) because of its ease of adjustment.

The number of taps required on the line depends on the nature of the filter we are attempting to synthesize. If the filter response is identically zero outside of some band, then an infinite number of taps are required, in principle. However, if the filter response is smoothly tapered over this band, then only a finite number of these tap weightings will differ appreciably from zero and a good approximation to the desired filter response can be had by utilizing a delay line with only a finite number of taps. The filter impulse response, including the band selection filter, can occupy at most a band of width W_F in frequency and approximately $NT_F = N/W_F$ in time. The time-bandwidth product of the filter impulse response can at most be equal to the product of these quantities which is simply N, the number of taps on the line. The practical design problems in obtaining a band selection filter, and the approximation problem associated with truncating the number of taps on the delay line will result in a filter impulse response whose time-bandwidth product is somewhat less than N.

The tapped delay line realization of the repetitive filter is valid at all frequencies from zero to infinity. In practice, the device would have a bandwidth limited by the frequency response of the delay line employed. If a band selection filter were used, it would probably be centered on the center frequency of the bandpass tapped delay line.

Sometimes it is desirable to use a tapped delay line whose center frequency is zero, i.e. a lowpass line. A number of commercially available distributed parameter electrical delay lines are of this type. In order to see how the desired response can be realized with tapped lowpass delay lines, let us write the impulse response h(t) in the form

$$h(t) = u(t) \cos \omega_o t - v(t) \sin \omega_o t$$

If h(t) is assumed to be bandlimited to the region $(-2\omega_o, 2\omega_o)$, then u(t) and v(t) will have a finite spectrum only in the frequency range $(-\omega_o, \omega_o)$. Noting the relations proved earlier,

$$\mathcal{H}\left[u(t) \cos \omega_o t\right] = u(t) \sin \omega_o t$$

$$\mathcal{H}\left[v(t) \sin \omega_o t\right] = - v(t) \cos \omega_o t$$

the desired impulse response h'(t) can be written

$$\begin{aligned} h'(t) &= \mathrm{Comb}_{T_F}\left[h(t)\right] - \mathcal{H}\,\mathrm{Comb}_{T_F}\left[\mathcal{H}\,h(t)\right] \\ &= \mathrm{Comb}_{T_F}\left[u(t) \cos \omega_o t - v(t) \sin \omega_o t\right] \\ &\quad - \mathcal{H}\,\mathrm{Comb}_{T_F}\left[\mathcal{H}\,(u(t) \cos \omega_o t - v(t) \sin \omega_o t)\right] \\ &= \cos \omega_o t\ \mathrm{Comb}_{T_F} u(t) - \sin \omega_o t\ \mathrm{Comb}_{T_F} v(t) \\ &\quad - \mathcal{H}\left[\sin \omega_o t\ \mathrm{Comb}_{T_F} u(t) + \cos \omega_o t\ \mathrm{Comb}_{T_F} v(t)\right] \end{aligned}$$

Now, if $\mathrm{Comb}_{T_F} u(t)$ and $\mathrm{Comb}_{T_F} v(t)$ were functions whose spectra were confined to the frequency range $(-\omega_o, \omega_o)$, the $\mathcal{H}$ could be brought into the bracket in the last term according to the rule. The Comb operator, of course, gives an infinite bandwidth, but suppose we consider a Comb operator which has been filtered so as to eliminate all but the very low frequency components. We shall denote this operator by Comb'_{T_F}. As a practical matter, the Comb operation which can be realized with a tapped delay line is limited in this way because of the finite bandwidth of the delay line. Recalling that the Comb operation involves multiplication in the time domain, the bandwidth occupied by $\mathrm{Comb}'_{T_F}\, u(t)$ is equal to the sum of the bandwidths occupied by Comb'_{T_F} and $u(t)$. Provided that the carrier frequency ω_o is chosen to be larger than the resulting bandwidth, $\mathcal{H}$ can be brought into the bracket according to the rule. Combining coefficients of $\cos \omega_o t$ and $\sin \omega_o t$, we have the simple result, ignoring constants of proportionality as usual,

$$h'(t) = \cos \omega_o t \; \mathrm{Comb}'_{T_F}\, u(t) - \sin \omega_o t \; \mathrm{Comb}'_{T_F}\, v(t)$$

Here the desired impulse response is repetitive in frequency, but this repetition extends only over the bandwidth of the delay line. As before, the Comb operations can be realized with the aid of tapped lines, where the tap weightings are equal to sampled values of the function in question. We denote $u_k = u(kT_F)$ and $v_k = v(kT_F)$. A method for realizing $h'(t)$ using tapped delay lines is shown in Figure 7. It can be easily shown that all time-varying terms in the impulse response of this system are zero. The configuration shown requires two lowpass delay lines, but no Hilbert transformers are necessary. If the band over which the frequency response should be repetitive has width W, the delay lines should have a bandwidth $\frac{1}{2}$ W.

Let us consider now how the tapped delay line network can be used in the synthesis of a linear FM pulse compression network with large TW product. The required group delay versus frequency characteristic is repeated in Figure 8. This characteristic can be resolved into the sum of the two time delay versus frequency characteristics shown in Figure 9.

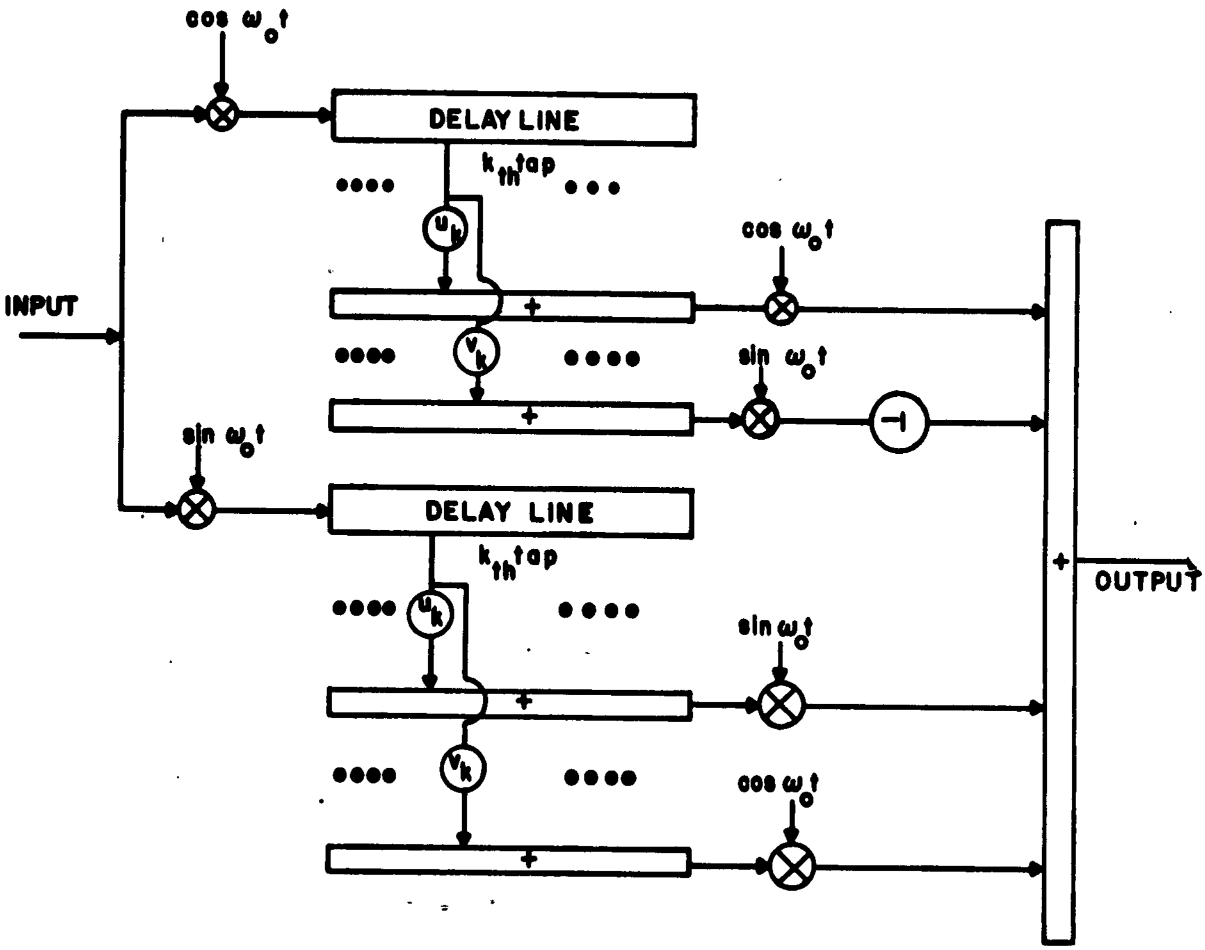
cos $\omega_o t$
DELAY LINE
k_{th} tap
u_k
v_k
INPUT
sin $\omega_o t$
-1
DELAY LINE
k_{th} tap
OUTPUT

FIGURE 7

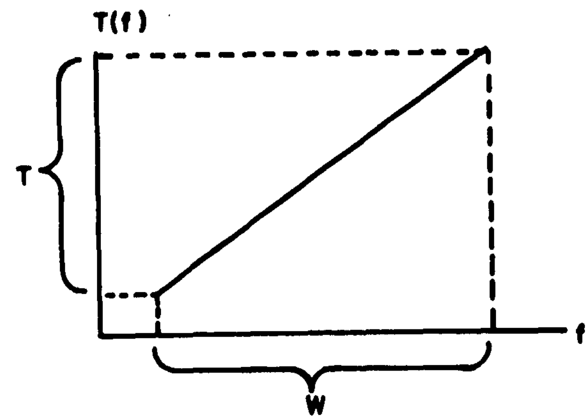

FIGURE 8

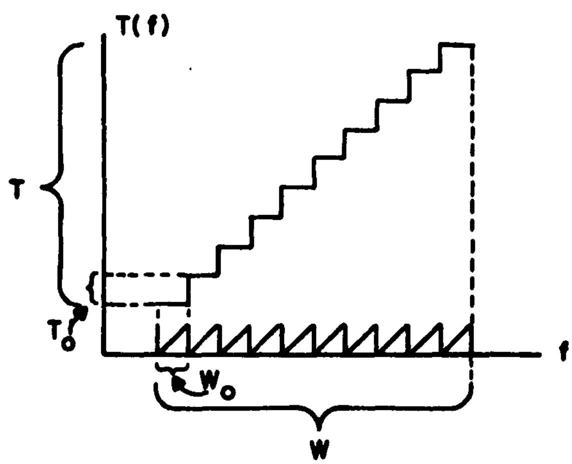

FIGURE 9

The sawtooth function is periodic in f and hence could be realized with the aid of a tapped line. The staircase function also suggests the use of a tapped delay line with uniformly spaced taps spaced T_o apart and with a band selection filter of width W_o at the output of each tap. Each of these networks would have to be all-pass, i.e. have uniform amplitude characteristic across the band, and the desired group delay characteristic would be obtained by cascading these two networks in series.

As a practical matter it is not possible to construct band selection filters which are perfectly rectangular and a repetitive filter which is a perfect sawtooth. One method which might prove satisfactory would be to use a set of amplitude selection filters, somewhat overlapping, to partition the signal band into a number of frequency bands, as shown in Figure 10. The output of each filter could be translated in frequency

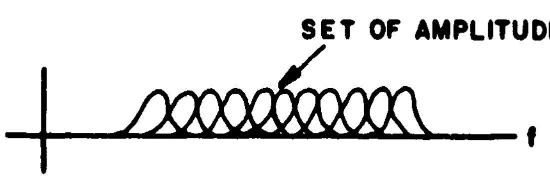

FIGURE 10

so as to obtain a set of equally spaced but non overlapping bands as shown in Figure 11. Each band can be delayed an amount proportional to its

FIGURE 11

frequency, the results added and passed through a repetitive network with the desired dispersive characteristic for each band. The result can be resolved back down into its separate frequency components with a set of band-selection filters, and the resulting bands can be translated so they once again occupy adjacent positions in frequency, as shown in Figure 10.

If the amplitude selection filters are reasonably tapered and reasonably closely spaced, the overall amplitude characteristic should be almost flat. The repetitive network need only have its nearly saw-tooth character within the bands where substantial signal exists and therefore the requirements imposed upon the repetitive network can be relaxed considerably.

Other arrangements can be used employing one or several repetitive networks which will allow us to obtain the desired dispersive characteristics over several frequency bands simultaneously. The above arrangement is meant to serve only as an example. The application of repetitive networks to the realization of this type of pulse compression system will require much further study.

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Roger Manasse

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